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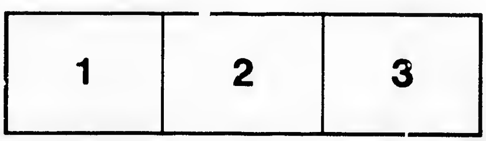

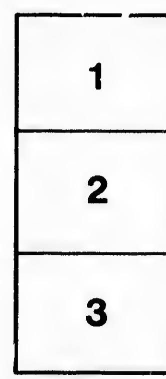

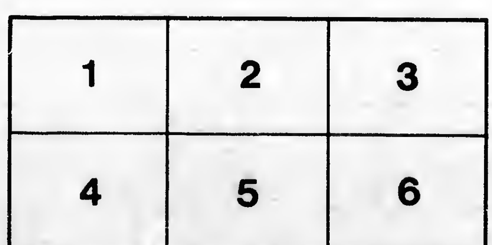

MEASUREMENT

OF

THE SUN'S DISTANCE.

BY

JOHN HARRIS.

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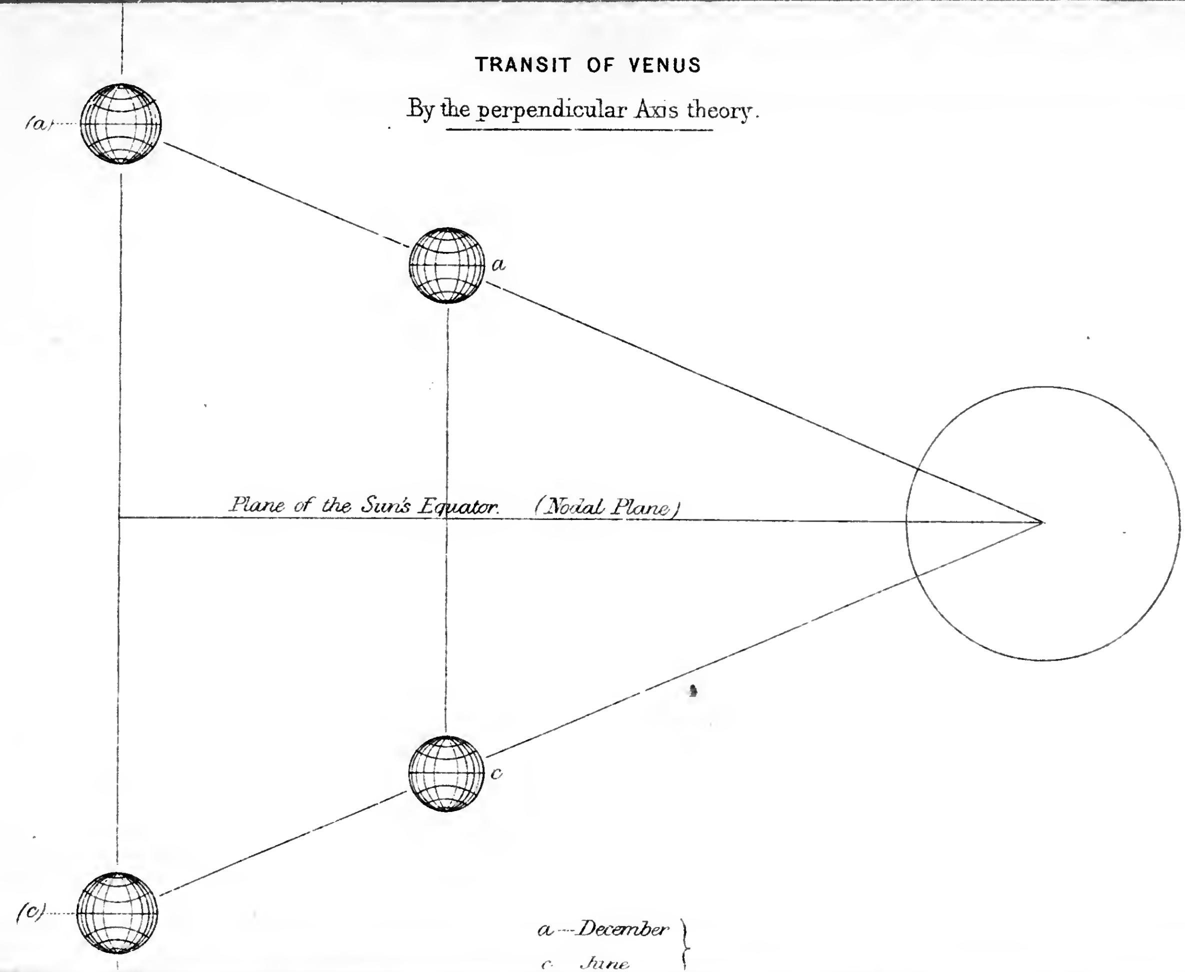
TRANSIT OF VENUS

By the perpendicular Axis theory.

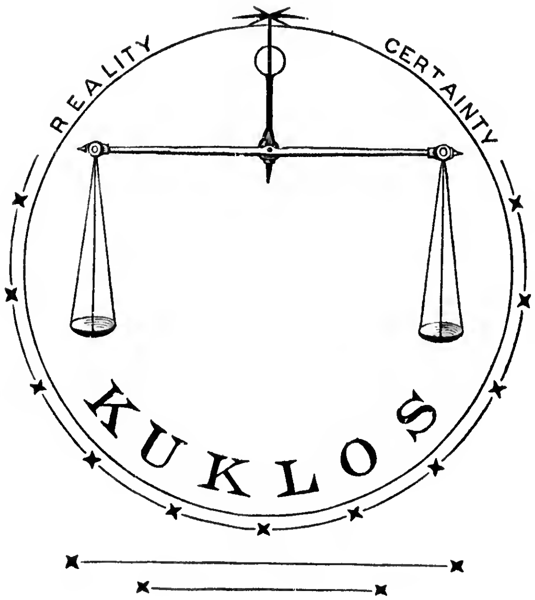


ASTRONOMICAL LECTURES.

MEASUREMENT
OF
THE SUN'S DISTANCE.

BY

JOHN HARRIS.



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SEPTEMBER, 1876.

LECTURE FIFTH.

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PREFACE.

This fifth lecture, or fifth division of the series to which it belongs, differs in one respect from those preceding it, inasmuch as we have not on this occasion to call in question or to condemn the present doctrine on the particular subject of which it treats.

It is true that, not long since, an error of about four million miles in the estimate of the sun's distance which had been accepted and agreed to by astronomers for many years previously, was discovered; and that it is only quite recently the correction of the error has been made and adopted. But this error was of the nature of a mistake which in combining the separate observations of several different observers was almost unavoidable, if it happened that any one of those observers, upon whose collective reports the general computation was based, had from misfortune, or want of due care, fallen into error and furnished a report which, being accepted as trustworthy and being in fact incorrect, vitiated the whole. Such a circumstance does not necessarily prove, nor in any degree evidence, the method itself to be unsound in principle or unreliable in practice. But it does afford evidence that the conditions necessitated by the particular method in question practically occasion a liability to error in the collective result, and it suggests very pointedly the desirableness of practical astronomy being in possession of some other method, or methods, equally reliable as to the sound theoretical character of the basis, and of such a nature in themselves that the individual observer, using due care and diligence, will be independent of the want of care or correctness on the part of other observers.

INDEX TO THE SEVERAL METHODS.

PLATES.

THE MEASUREMENT

OF

THE SUN'S DISTANCE.

The methods at present known in practice, and which have been utilized with more or less success for the purpose of ascertaining the distance of the sun from the earth are two, that of geocentric parallax (or zenith meridional observation), and the transit of the sun's disc by the planet Venus.

Our present purpose is to suggest, explain and illustrate, so far as may be necessary for the elucidation of this preliminary explanation, certain methods which we believe to be essentially new and hitherto unpractised.

(1.) If, by direct observation, the earth's velocity of revolution in its orbit relatively to the velocity of its rotation can be ascertained with certainty and precision, such information, together with the data already known, will enable the distance of the sun from the earth to be readily computed.

The mode of observation by which we propose to obtain this information is by utilizing the retrograde or backward movement of a station on the equator in its rotation round the earth's axis, when on that side of the earth nearest to the sun, compared with the advance of the earth itself in its orbital revolution during the same time. To explain the proposed mode of proceeding, let us suppose the station of observation to be situated on the equator and that the earth's rotation through 60 degrees is to be subjected to the comparative observation.

Knowing the time at which the meridian of the observatory will pass the sun, the observation is to commence two hours before that time. The situation of the sun as seen from the earth's centre, and as seen from the station on the equator, is to be recorded, and also the exact time of the sun's centre transitting the meridian of the observatory. And, again, the situation of the sun as seen from the earth's centre, and from the observer's station, four hours later than the time of the first observation, is to be carefully determined.

Having thus ascertained by direct observation the parallactic displacement of the sun consequent upon the compound motion of the observatory, we shall be able to deduce the linear velocity in miles of the earth in its orbital revolution around the sun, because we already know the linear value in miles of the arc through which the retrograde or reverse motion of the earth's rotation has carried the observatory, and we have ascertained the value of the *chord* of that arc as a part of the orbital circle of the earth's annual revolution. We know also in time and in angular measurement the quantity of orbital arc traversed by the earth during the observation. For example: the observation being made for 60 degrees of the earth's rotation, we will assume that 9′ 43″ is the parallactic displacement of the sun between the first observation and the last. Now the orbital arc moved through by the earth in 24 hours is a little less than one degree, viz., 59′ 10″, so that the one-sixth of this quantity, viz., $9' 52''$ is the displacement of the sun caused by the earth's orbital progress, and which would be the difference between the two observations, if both were made from the earth's centre. But the actual displacement shown by the direct observations from the station on the equator is (by the supposition) $9' 43''$. The $9''$ of difference is therefore due to the reverse or retrograde movement of the station in consequence of the earth's rotation. This $9''$, however, represents the chord of the terrestrial arc, and not the arc itself. Now it is the terrestrial arc which we must compare with the solar arc to obtain the linear velocity of the one from the known linear velocity of the other. Therefore, as $9'43'' : 9''.427 :: 62 : 1$. That is, the linear velocity in the orbital revolution of the earth itself is 62 times that of the station on the equator due to the rotation of the earth. Now since if the linear velocities were equal the angular velocities would inversely measure the comparative lengths of the radii, we should have accordingly $360° : 59' 10''$ * as the proportion of the greater length which the sun's radial distance would have, compared with the radius of the earth, if the linear velocities were equal. But the linear velocity of the earth's orbital motion being determined as 62 times greater than that of the equatorial surface due to the earth's rotation, the computation, taking the radius of the earth at 4,000 miles, will be $4{,}000 \times 365 \times 62 = 90\frac{1}{2}$ millions of miles as the sun's distance from the earth.

The same observation also directly furnishes the geocentric horizontal parallax of the sun. For, taking the preceding example, since the chord of the arc of $60°$ equals the radius of the circle, the difference between the

* Or 365 : 1.

observed angle of parallactic displacement and 9′ 52″ (which difference we have assumed in the foregoing as 9″) is the geocentric parallax of the sun. Therefore, from this quantity, which results immediately from the observed displacement of the sun, and from the known magnitude of the earth and velocity of the earth's rotation, the distance of the sun from the earth can be readily determined in the usual manner.*

(2.) By the angle of the moon's illumination.

In the accompanying figure (fig. 1), the moon is represented at A, in quadrature ; that is, in the situation relatively to the sun and earth which she occupies when the one-fourth of her orbital revolution is completed. Now in her position at A, when so situated, rather more than the one-half of the moon's disc, as viewed from the earth, is illuminated by the sun, which is obviously a consequence of the sun's light striking the moon at an angle, with a line joining the centres of the earth and moon, rather less than a right angle. For if, as shown at B, on the other side of the figure, the moon be so situated in her orbit that the direction of the sun's rays forms a right angle with the line joining the centres of the earth and moon exactly, one half only of the moon's hemisphere will be illuminated.†

* The number of seconds contained in the circle are 1,296,000, which, divided by 9 = 144,000 ; which, multiplied by 4,000 miles (the length of the earth's semi-diameter) = 576,000,000 as the circle of the earth's orbital circle. The distance of the sun from the earth, which is the radius of that circle, equals, therefore, 91½ million miles.

† To simplify the explanation we are here leaving out of consideration, for the moment, the great comparative magnitude of the sun, and consequent extension of the illuminated surface of the moon beneath the equator.—*See* page 14.

SUN'S DISTANCE

By the Angle of Moon's illumination.

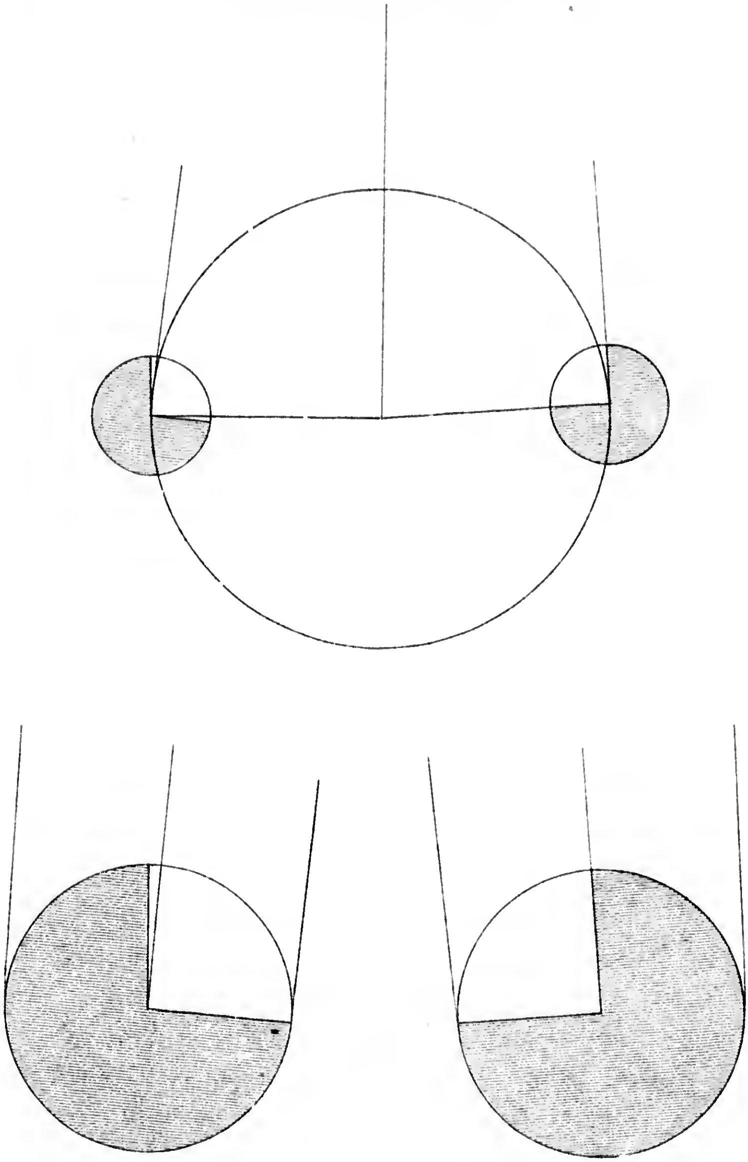

Fig. 1.

The moon's distance from the earth has been long since ascertained by means of geocentric parallax, and it may be assumed that the distance is now known with an approximation to precision. Since a ray of light from the sun to the moon is equivalent to a line joining the sun and moon, the careful astronomical observation of the angle of the moon's illumination at quadrature (by measuring the magnitude of that part of the moon's hemisphere illuminated in excess of the semi-hemisphere), will furnish the angular distance of the moon from the earth in terms of the earth's orbital circle; or, in other words, it will determine an arc of that circle in linear measurement equal to the distance between the earth and the moon, of which distance the metrical value in miles is already known. But so soon as the value of a circle, or of any definite fraction of the perimeter of a circle, in terms of the linear metrical standard, is ascertained, the length of the radius in terms of that standard becomes known. Therefore the distance of the earth from the sun, which is the radius of the earth's orbital circle, will become correctly known so soon as the angular illumination of the moon at quadrature has been carefully measured, and accurately determined.

To measure the angle of illumination it is not necessary, however, that the moon should be at the place of quadrature. By observing with exactitude the angular situation at which 16′ more than the one-half of the moon's hemisphere is illuminated, the difference between that angle and quadrature will furnish the angle subtended by an arc of the earth's orbital circle contained in the intervening space, of known metrical value, between the moon and the earth.* For example, let us suppose in fig. 2, the moon to be at that place A of its orbit, where a line joining the centres of the earth and moon is exactly at right angles to a line joining the centres of the moon and sun. We have first to consider that the diameter of the sun as seen from the earth subtends an angle of 32′; and since the moon, at that place in her orbit, is at the same distance as the earth from the sun, the angular magnitude of the sun's diameter as there seen from the moon will be the same. Consequently, since the moon's entire diameter as seen from the sun subtends an angle of only about 2″, the sun's rays, impinging upon the moon's globe in a converging cone, will strike about 16′ beyond the central circle which, posited horizontally to a line joining the centres of the sun and moon, divides the moon's globe into equal hemispheres. Viewed, therefore from the earth, the moon will appear illuminated to an angular distance of 16′ below the equator, or, in other words, the whole of the moon's upper hemisphere and 16′ of the lower hemisphere will be illuminated by the sun's light. Now let the moon move onwards to the place of quadrature at B. We will assume that astronomical measurement determines the angle A E B as 9′. It follows that 16′+9 =25′ of the moon's dark hemisphere will now be illuminated. And, because the line S A is perpendicular to the line A E, and the line S E perpendicular to the line E B, it follows that the angle E S A also contains 9′ of arc, which arc belongs to the circle of the earth's

* Other situations of the moon may be made available for the same purpose, only that the more directly the required data are furnished by the observation, the more correct, *cæteris paribus*, will be the result.

SUN'S DISTANCE

By the Angle of Moon's illumination.

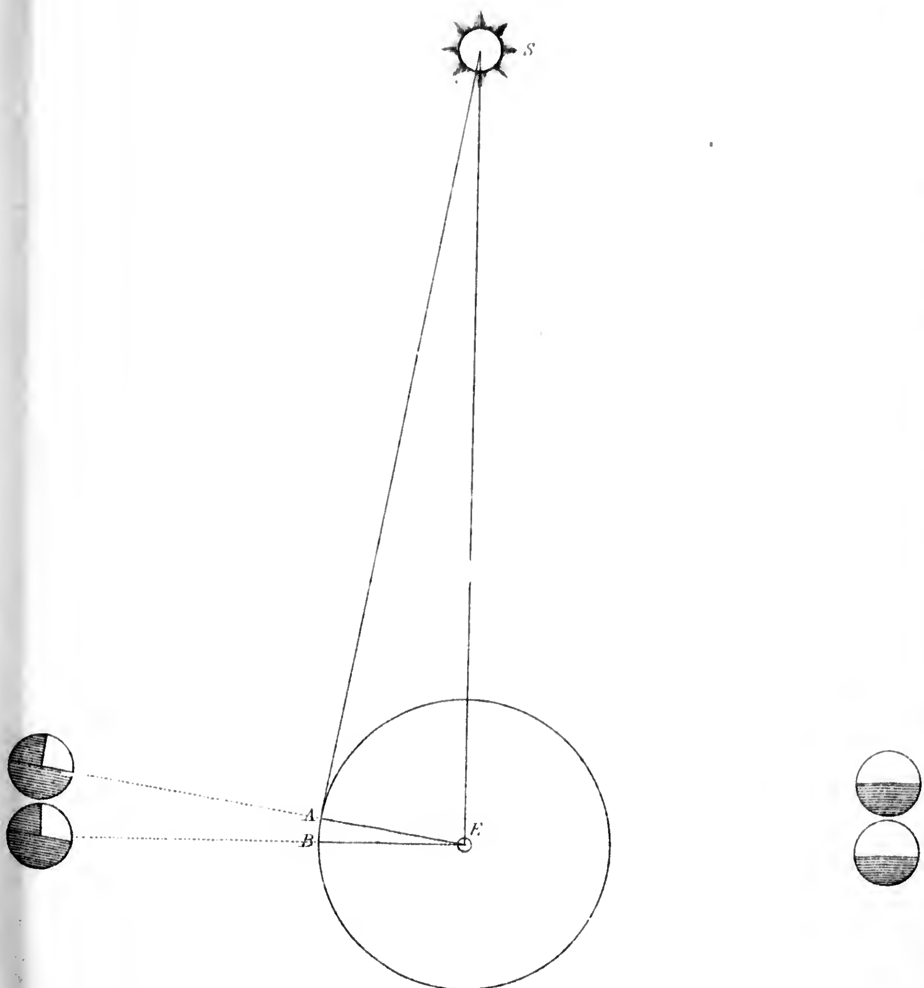

Fig. 2.

orbital revolution and is known to equal in metrical value 60 times the radius of the earth. Therefore, taking the earth's radius as before at 4,000 miles, we have 9′ of the earth's orbital circle, equal to 240,000 miles, which furnishes the radial distance of the sun from the earth as nearly 92 million miles.*

(3.) In consequence of the earth's progressive orbital advance during the time occupied in its diurnal rotation, the earth having completed a siderial rotation has to overtake the sun by a space which is a known definite fraction of the circle bounding the earth's sphere. Similarly in the moon's revolution around the earth, the arc of difference between the siderial and synodic revolution is a known definite fraction of the moon's orbital circle (as the earth's satellite). By ascertaining the time occupied by the earth with the velocity of its orbital revolution in moving through the same arc, the distance of the sun may be ascertained; or, in other words, if we can ascertain the linear value of this differential angle compared with the similar angle of the earth's orbital circle, of which it is a consequent (and with which angle it is necessarily equal) the distance of the sun will become known. Now the moon itself as seen from the earth subtends an angle sufficiently large to admit of very accurate appreciation as a definite fraction of its own orbital circle, and of the differential angle belonging to that circle, of which (diff. angle) the value in terms of the earth's orbital circle is required. If, therefore, we can measure the value of the moon's diameter in terms of the earth's

* 91,680,000 miles.

orbital circle, we can therefrom compute the value of the differential angle, and hence obtain the sun's distance. The conditions of this method may also be stated as follows: since we know the time occupied by the moon in completing a revolution around the earth, and we know the fraction of that orbital circle contained in the moon's disc, if we ascertain the time required by the earth with the velocity of its orbital revolution around the sun to pass through the angle subtended, at the distance of the moon, by the moon's diameter, we shall thereby obtain knowledge of the comparative linear velocity of the earth around the sun, to that of the moon in its orbit around the earth; from which data we can compute the sun's distance.

To ascertain the time occupied by the earth in passing through an arc equal to that fraction of the circle of the moon's orbit, made apparent to us and defined by the apparition of the illuminated moon, as the angle subtended by the moon's diameter, an occultation of the sun by the moon (fig. 3) affords the most favourable opportunity. The centre of the sun, if the occultation be such that the centre of the moon will pass over the sun's centre, or any clearly defined spot so situated on the sun's disc that the equator of the moon will pass over it, will equally well answer the purpose of the observation, which is in the first place to ascertain the apparent time occupied by the earth in passing through a fraction of its orbital circle equal to the angular value of the moon's diameter. Now the apparent time thus observed would be the actual time of the earth's velocity if the moon were at rest; but, in fact, as the earth in its solar

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VELOCITY OF EARTH'S MOTION

Measured by Moon's transit of Sun's Centre

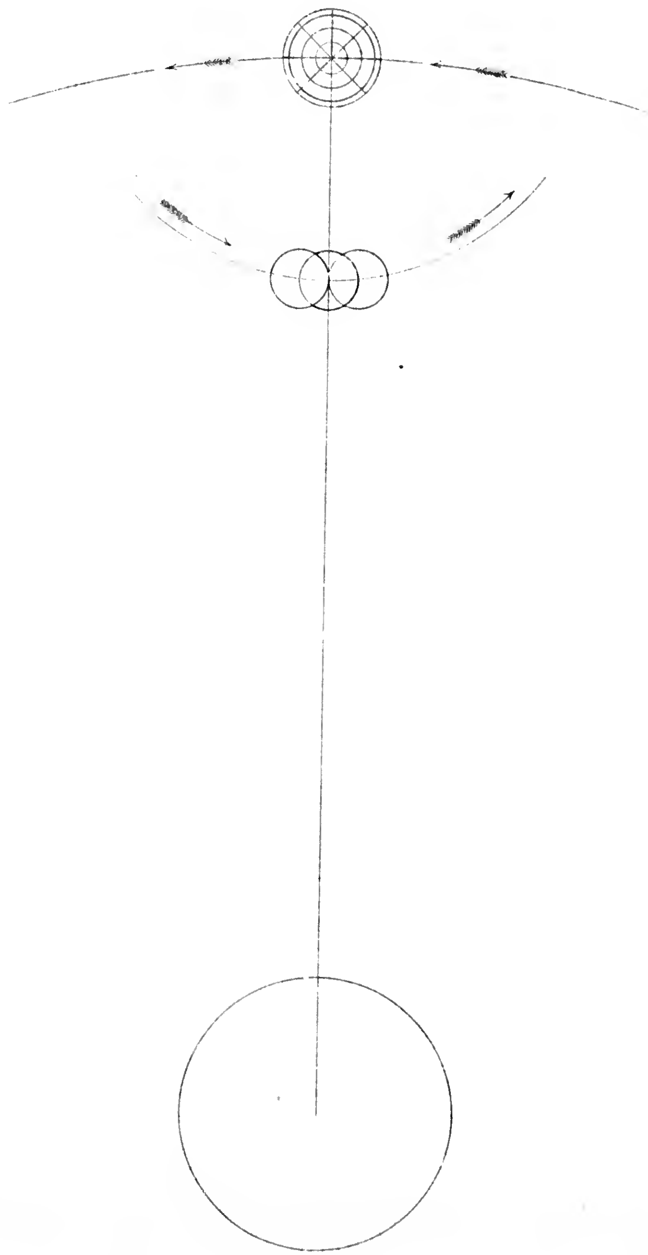

Fig. 3

SUN'S DISTANCE

By the lunar differential Method.

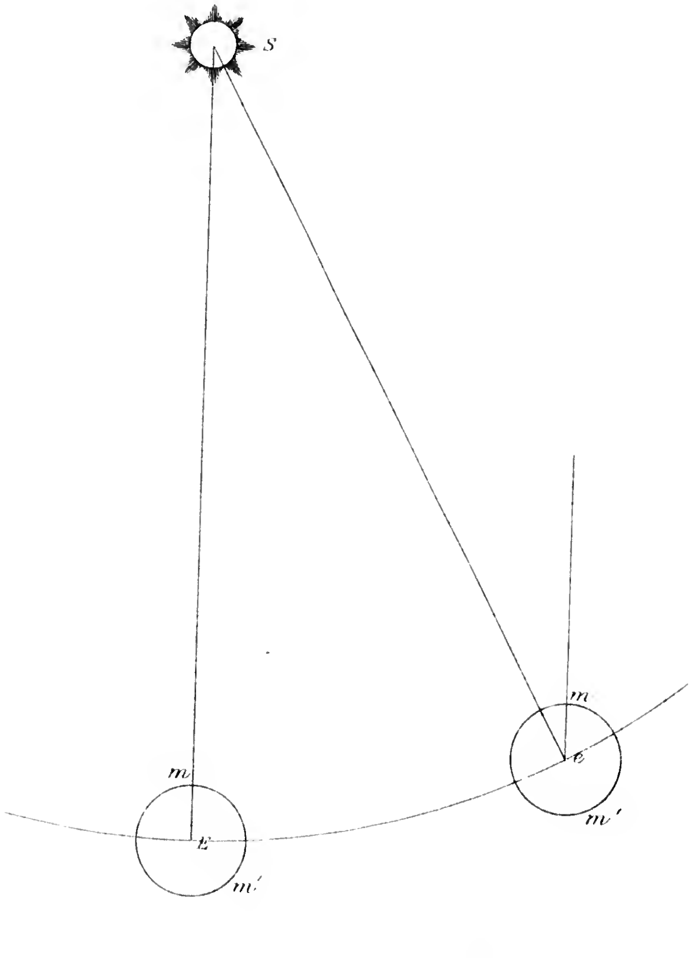

Fig. 4.

orbit is moving with a certain velocity in one direction, the moon in its (terrestrial) orbit is moving, with a lesser velocity, in the opposite direction. Therefore, since the moon, instead of remaining at rest for the earth to pass it, has, in part, taken itself out of the way, the apparent time required by the earth to pass through the arc of the lunar circle equal to the moon's diameter is, by so much, less than the actual time. The velocity of the moon's motion in its orbit is known, and we can estimate with precision the time required by the moon to pass through an arc of its orbital circle equal to its own diameter. To deduce the actual time of the earth's velocity in passing the moon from the apparent time of the observation, we have, therefore, to *add* to the observed time a fraction proportional to the time which the moon requires to pass through an arc of its orbit equal to its own diameter. As an illustration, let us assume the observed time of the passage of the moon's diameter across the sun's centre, or over the solar spot, to be 1′ 56″. Now the time required by the moon to pass through the arc of its own diameter is, by computation, 59 minutes. Therefore, to 1′ 56″ we have to add 1′ 56″ ÷ (59 ÷ 1·94); so that the actual time comes out very nearly 2′. Referring to the figure (fig. 4)—in which *S* represents the Sun, *S e* the solar radius-vector of the Earth, *m m′* the Moon's orbit, and *E*, or *e*, the Earth—the angular situation of the moon and earth in relation to the sun after 28 days' orbital progress, is shown on the right of the figure. The moon having then completed one sidereal revolution, and the earth having completed the thirteenth part of her orbital circle, we have the arcs subtending respectively the two equal angles, *E S e* and *S e m*, proportional to each other in the ratios of the distances each to the other of their respective centres of revolution—to wit, the distances of the sun from the earth and of the earth from the moon. Now since the earth's orbital velocity carries it through 32 minutes of the moon's orbit in 2 minutes of time, the time occupied by it in moving through the differential arc of 27° 41′ (of the moon's orbit) would be 104 minutes 5 seconds. But the time occupied by the earth's velocity in moving through the greater arc (27° 41′ of its own orbit) is 28 days. Therefore the greater arc is proportional to the lesser as 387½ to 1. Consequently the sun's distance is 387½ times that of the moon from the earth, and, taking the moon's distance as 238,800 miles, the sun's distance equals about 92½ million miles. The same result may be also arrived at by inversely estimating from the relative velocities. For, taking the proportion of a very little less than 2′ to 59′, as 29⅔ to 1, since the angular velocity of the moon is 13 times greater than that of the earth, we shall obtain the proportion of the greater distance if we multiply 29⅔ × 13, which gives 386 times the radius of the moon's orbit for the distance of the sun from the earth (equal to about 92 million miles).

(4.) By the ascent and descent of the earth in its orbital revolution. This method consists in choosing three stations, one of them on the equator, and of the others—one in high latitude in the northern hemisphere, and one in similar high latitude in the southern hemisphere. The longitude of the stations to be respectively such that each, when passing through the plane of the solar equator, will have the sun on its meridian. The vertical distance between these stations (measured as the chord of the vertical arc joining three places having the same longitude in common, and, respectively, having the same latitude as each of the stations of observation) being known, it is required to determine, by observation, the exact place of the sun in the ecliptic at the time when the meridian of each station successively transits the centre of the sun. We then have the vertical quantity contained in a definite small angular section of the ecliptic measured in the known metrical value of the vertical linear distance between the terrestrial stations. Hence the linear value of the earth's orbital circle and therefrom of the sun's distance becomes readily determinable. Such observations would be preferably made near the time of the equinox, or when the sun is not, at most, more than two months from one of the nodes. And it is to be observed that, since the earth's vertical velocity is greatest at and near to the time of crossing the nodal plane of the sun, and for a brief period at each of the solstices becomes nothing, the quantity directly obtained by this method would have to be rectified accordingly, in order to get the average of the vertical motion throughout the entire orbit or semi-orbit. Such rectification, however, the angular velocity and magnitude of the angle being known, and the nature of the orbit, as a circle or ellipse posited obliquely, being apprehended, would present no difficulty.

To illustrate this method, the following considerations may be stated:—45° of the earth's vertical motion combine with and occupy the same time as 180° of the earth's horizontal motion, *i.e.*, the vertical motion is to the horizontal as one to four.

The circle of the earth's rotation equals by time very nearly one degree of its orbital horizontal motion.

One degree of the earth's orbital horizontal motion equals by time $\frac{1}{4}$ degree of its vertical motion.

If, therefore, it be found that the entire vertical angle between the two extreme stations of the northern and southern hemispheres, which we will suppose to be 4,000 miles apart (measured as the chord of the vertical arc joining the latitudes of the two stations), contains 9 seconds, then, since the earth's horizontal orbital motion will occupy about $3^m 40^s$ in moving through 9″, the vertical motion will occupy $14^m 40^s$ in moving through the same angle. But $14^m 40^s$ represents about one-hundredth of the earth's circle of rotation, therefore the longitude of the two extreme stations would require to be such respectively that they would be 3° 36′ apart. The meridian of the equatorial station would have such longitude as to be situated equidistantly between the meridians of the two extreme stations, 1° 48′ from each.* (See fig. 5.)

Instead, however, of thus confining the linear measuring distance to 4,000 miles of the earth's diameter, 12,000 miles might be made available for the purpose. Reference to the figure (fig. 5) will, it is thought, make the manner of the intended application sufficiently intelligible for the present purpose—viz., by taking the extreme stations *c* and *e* in such high latitudes, north and south respectively, as to be 6,000 miles apart (measured by the chord of the vertical arc joining the

* In strictly computing the required difference of longitude, an allowance would have to be made for the onward progress of the earth in its orbit.

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SUN'S DISTANCE

By the Vertical Angle of Earth's descent.

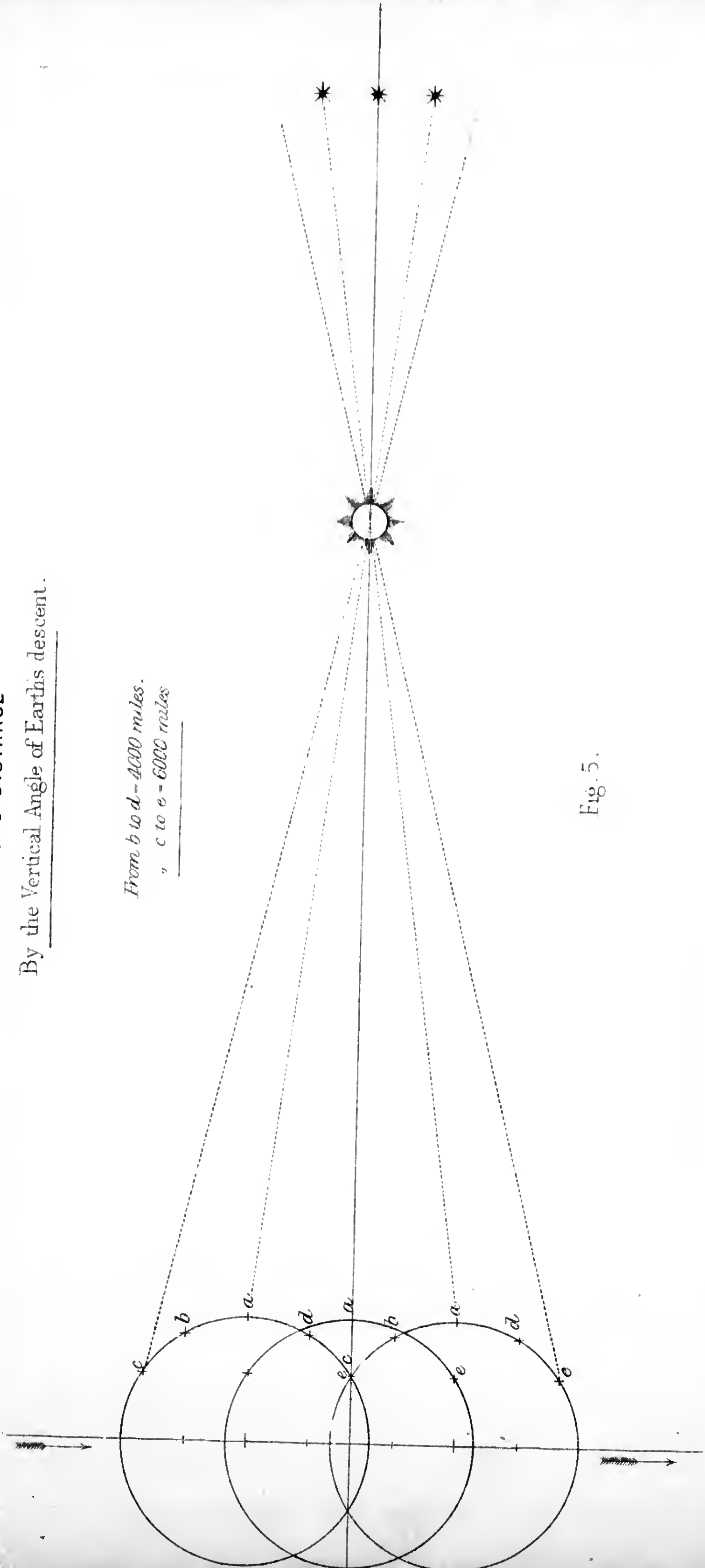

Fig. 5.

latitudes, as before.) . . Commencing the compound observation when the southern station reaches the nodal plane of the sun's equator, and completing the observation when the northern station reaches the same plane. The parallactic displacement of the sun would thus furnish the measurement of a definite sectional arc of the ecliptic. The distance of longitude between the meridians of the stations, as determined by computation, may be rectified by repeated experiment (*i.e.*, by shifting the localities of the stations as required) until an indefinite approximation to accuracy is obtained.

(5.) A method of measuring the sun's distance quite distinct from the foregoing, but allied thereto, because dependent upon the vertical motion of the earth in its orbit, may be described as characteristically consisting in the astronomical observation of the phenomena called solar spots, which are sometimes seen to traverse the sun's disc. It is well known that at two seasons of the year only, namely at the summer and winter solstices, in June and December, are the spots seen to cross the sun's disc horizontally. At other seasons of the year, the path of the spot in crossing the sun is oblique, either ascending or descending as the season is approaching the summer or the winter solstice. It has been already pointed out elsewhere that the apparent obliquity of the paths of the solar spots harmonizes perfectly with the perpendicular axis theory, and and may indeed be considered to constitute a part of the demonstration of the truth of that theory. The apparent obliquity is, we consider, certainly an effect of the vertical orbital motion of the earth combined with its orbital horizontal motion. To apply this method, however, we must assume that the velocity of the spots in traversing the circle of the sun's equator has been correctly ascertained, or, to speak with more particularity, that the velocity of the sun's rotation, by which the spots are carried around it equatorially, has been so ascertained. Then since such rotation, assuming it to exist, must be certainly in the same direction as that of the earth's orbital revolution, the apparent motion of the spots in circulating around the sun, as seen from the earth's centre, must be the resultant of the difference in the velocities of the two motions, for if the velocity of the sun's rotation were such that a spot on the solar equator was carried around with the same angular velocity as that of the earth's orbital revolution, the appearance to the terrestrial observer (from the earth's centre) would be that of a spot motionless and constantly occupying the same situation on the sun's surface. Now if the angular velocity of the sun's rotation were so great as one complete rotation in 24 days, this velocity being 15 times greater than that of the earth's orbital revolution, and the velocity of the earth's vertical motion being only one-fourth of its horizontal orbital motion, the obliquity produced in the apparent motion of the spots across the sun's disc would be scarcely appreciable, because the deviation from a perfectly horizontal plane would be so small in amount. But if we reflect that in consequence of the circumferential curvature of the sun's globe (as of every other globe) not more than, at most, about 90 degrees of the hemispherical surface would present the spot at such a visual angle as to be visible from the earth, it will become apparent that an

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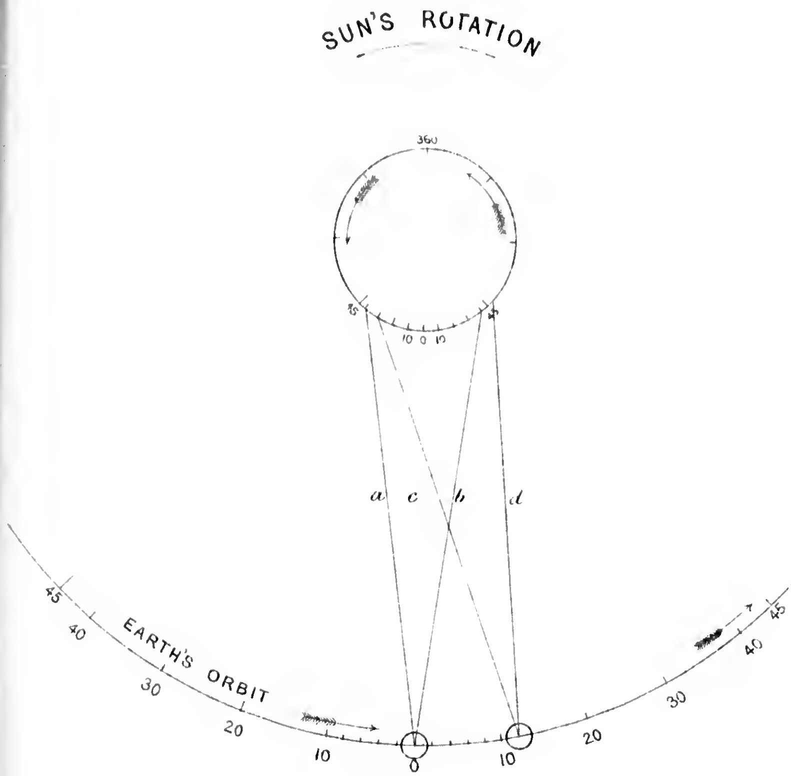

Fig. 6.

angular velocity of rotation by the sun, equal to about six times that of the earth's orbital motion, would bring the spot into the field at the one side of the sun's disc, and take it out of the field at the opposite side, within about twelve days; because, beyond the limit of the 90 degrees, the foreshortening angular effect would become so great as to present the spot almost edgewise, and so render it invisible, whether approaching on the one side or receding at the opposite.

This interpretation of the facts may become more readily appreciable by aid of the accompanying figure (fig. 6). In twelve days, which is about the average or most usual time that a spot remains visible, the earth will have advanced nearly 12° in its orbit, that is, in the same direction in which the rotation of the sun, or of the sun's surface, carries the spot. This advance will evidently occasion an extension of the solar arc throughout which the spot will be visible; so that if, supposing the earth had remained stationary in its orbit, the visual solar arc would have been about 78°, it will have extended to about 90° in consequence of the orbital advance of the earth. Hence, to complete the circuit of the sun's equator, the spot will occupy 48 days (say about 50 days), which may be considered to measure the angular velocity of the sun's axial rotation. It is not, however, to be inferred in such assumption that, supposing the spot to remain existent and unchanged, it will reappear at the sun's eastern limb at the expiration of about 36 days. Such inference would overlook the continuance of the earth's orbital motion, which, in that time, would add nearly 36° to the 270°, making a total of 306°, throughout which arc, equivalent to about 41 days in time, the spot would be occulted by the sun's globe.*

One of the most distinctly observed phenomena belonging to the solar spots is the apparent obliquity of their paths across the sun's disc, with exception of two semi-annual periods in the year when those paths form straight lines. Another observed phenomenon is that the direction of the angle of obliquity during five months of the year is inverted during the other five months. Now this apparent obliquity is, we consider, certainly attributable to the vertical ascent and descent of the earth's orbital path. But if such be the cause, it follows (1) that the angle of obliquity of the path will necessarily measure the angular velocity of the spot in its revolution around the sun, and therefore of the sun's rotation; and (2) that the obliquity of the path of the spot, *i.e.*, the amount of its deviation from a horizontal line, may be utilized as a means of measuring the sun's distance in diameters of the sun, for the vertical amount of that deviation is parallax of the

* The most direct and probably the best method of deducing the period of the sun's rotation from the observed velocity of a solar spot would be simply to determine accurately the angular progress of the spot, as seen from the earth's centre, when near the central part of the sun's disc during one complete rotation of the earth, *i. e.*, during 24 hours. Now the earth, during the 24 hours, will have moved round the sun in the direction of its rotation nearly one degree (·9863 of a degree); therefore the observed angle, with addition of this terrestrial quantity, will be that part of the circle of the sun's equator moved through in its rotation during 24 hours. The time, therefore, occupied by a complete rotation of the sun will be simply 24 hours multiplied by the number of times the observed angular quantity increased by addition of the angle moved through by the earth, is contained in 360°.

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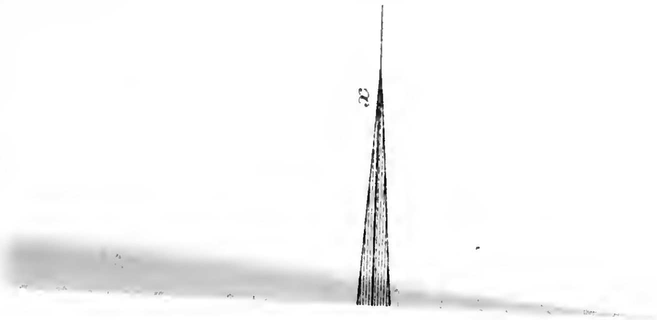
SUN'S DISTANCE

By the Angle of Incidence [illegible]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spot as projected on the sun's disc, occasioned by the ascent and descent of the earth in its orbital revolution.*

Let us suppose the observed parallax of the spot, taking the extreme limit of displacement, north and south, on the sun's disc to amount to 13 minutes: we shall then have the proportion..As 13′ : 47° : : the Sun's radius: the number of times that radius is contained in the radius of the Earth's orbit. For the purpose of illustration we will assume the sun's radius at 420,000 miles. We then have $420,000 \times 217 = 91$ millions of miles as the radius of the earth's orbit or distance of the earth from the sun.

(6.) A method of measuring the sun's distance quite distinct in character from the preceding—for the preceding methods are all dynamical in character, whereas the method now about to be described is statical in character—may be termed the geometrical method. In it a knowledge of the sun's distance is attained by observing the relative angles subtended by the diameters of the sun, the earth, and the moon, respectively, when seen directly or indirectly from several points of view.

In Fig. 7, let S represent the sun; E the earth; and MM' the moon at the two opposite extremities of its orbit, *viz.*, at conjunction and at opposition. The sun's light

* Supposing the spot to remain permanent or unchanged for some considerable time, and that the angular velocity of the sun's rotation were only the same as that of the earth's orbital revolution, the effect would then be that the spot would, to the terrestrial observer, appear to ascend and descend vertically on the sun's disc throughout an angle of $23\frac{1}{2}°$ beneath and $23\frac{1}{2}°$ above the solar equator, *i. e.*, throughout an angle of 47° (45°).

shining past the spherical earth, projects its shadow as a dark cone to the point *x*. The angle *a x b* is therefore the angle subtended by the sun's diameter as seen from the point *x*. Now since the linear value of the earth's diameter is known, and the distance of the moon from the earth at opposition is known, and the breadth of the shadow, at *g h*, is ascertained from the time occupied by the moon in passing through it,—the distance *E x*—(*i.e.*, the length of the earth's shadow)—becomes also known; consequently the value of the angle *c x d*, or *a x b*, which is the same, is known. Since the point *f* is on the surface of the earth, from which the astronomer views the sun, the angle *a f b* is the observed angle subtended by the sun's diameter. It is at once evident that the angle *a f b* is greater than the angle *c x d*; consequently if the lines *x c* and *x d* be produced indefinitely, they must eventually intercept the lines *f a* and *f b* in the points *a* and *b*, at the two extremities of the sun's diameter. If, therefore, we ascertain by observation the exact value of the angle *a f b*, at the time the moon is in opposition, we shall have the means of readily computing the distance *S E* of the sun from the earth. To illustrate this by example, we have the distance *E x* of the earth's shadow ascertained to be equal to 218 semi-diameters of the earth, and, hence, the value of the angle *E x c* is determined as 15′ 46″. (Now the angle of the sun's diameter, as observed at different times of the year, is supposed to vary from 31′ 32″.0 to 32′ 36″.2, but, with respect to an actual difference in the distance of the sun, we must either decline to accept this reported great variation, as correct only in respect to an apparent variation in the sun's magnitude when viewed at different seasons from the same locality, or from places situated nearly in the same latitude, and presumable erroneous in fact, in respect to observation made from the earth's centre through an atmosphere at all seasons in the same condition as to temperature, density, and humidity; or, otherwise, we must accept the assumption that the earth, in the course of each annual revolution, approaches and recedes from the sun through a space equal in linear extent to nearly six times the diameter of the moon's orbit.) For the present purpose we will assume the average angular value of the sun's semi-diameter to be determined as 15′ 55″. The difference between this angle (15′ 55″) and the angle $E x c$ (15′ 46″), which equals 9″, is the angle subtended by the earth's semi-diameter as seen from the sun, and, therefore, since the actual length, or linear value in miles, of the earth's semi-diameter is known, we have ascertained the value of a definite fraction of the earth's orbital circle, and hence the distance of the sun from the earth, which is the radius of that circle, becomes known. For instance, if 9″ be the ascertained difference between the angles, then, since 1 degree contains 400 times 9″, multiplying 400 by 4,000 miles as the linear value of the earth's semi-diameter, we obtain 92,800,000 miles as the distance of the sun from the earth.*

This method furnishes the angular value of the earth's diameter as seen from the sun, which is already obtained with an approximation to correctness by means of the geo-

* Taking the value of the earth's radius at 3,950 miles, which is very nearly the actual estimate, the distance will be 91,640,000 miles. These computations are, however, mainly intended to illustrate the methods.

centric parallax of the sun. It is a question whether or not this method now proposed is susceptible of a greater degree of exactitude. As an entirely independent and distinct method, however, it cannot fail to possess some considerable degree of interest and utility.

Instead of measuring the angle of the shadow behind the earth, the same angle may be indirectly obtained by measuring the breadth of the sun's light at any known definite distance between the earth and the sun. This may be readily explained by reference to the figure; for instance, if the moment when the limb of the advancing moon commences to interpose itself between the sun and earth on the one side be exactly determined, and also the moment of the conclusion of the egress of the moon on the other side of the orbit, so as to ascertain the exact time occupied by the moon in traversing the angular space of the sun's diameter (viewed from the earth), the required value of the angle at the distance of the moon's semi-orbit would become known. More favourable for this purpose would be a transit of the sun by one of the inferior planets. Records of carefully observed transits of Venus might be made available perhaps to determine in this manner the precise value of the angle of the earth's shadow, supposing the same records to include the angular value of the sun's diameter as seen at the time of the transit.

(7.) A method, which may be considered as allied to the preceding, consists in measuring by astronomical observation the apparent value of the sun's diameter, as seen, on the one hand, from a station on the earth's surface when the sun's centre is over the meridian of that station;

SUN'S DISTANCE

By the differential Angle of Earth's Radius.

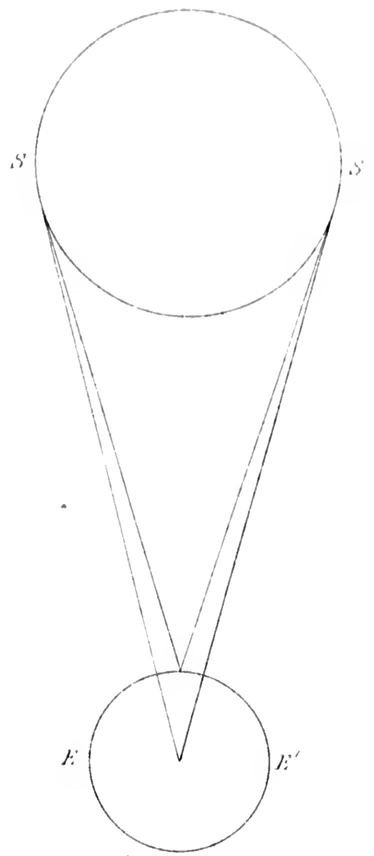

Fig. 8.

and, on the other hand, as seen, at the same time, from the earth's centre.

The manner in which it is proposed to obtain the comparative angles, is by means of the transit instrument, by which, having first ascertained with precision the angle subtended by the semi-diameter of the sun when the sun's centre is over the meridian of the station, the time elapsing until the extremity of the sun's diameter (*i. e.*, the edge of the sun's limb) is over the station, is to be carefully observed, which time will measure the angle subtended by the semi-diameter of the sun as viewed from the earth's centre. This last angle must be evidently less than the former, and by the difference the distance of the sun may be determined, because the metrical value in miles of the earth's semi-diameter is already known, and if the two lines, the inner of which has a greater obliquity than the outer, be produced until they eventually meet, the point of interception will be the sun's distance, and must be directly proportional to the semi-diameter of the earth, in a ratio determined by the observed angle.*

The principle of this last method is fundamentally the same as that explained in the case of the earth's shadow, and it is possible that the convenience, directness and simplicity of this method, notwithstanding the delicacy and extreme accuracy of observation requisite, may render it preferable and practically more advantageous

* Instead of the semi-diameter, the entire diameter of the sun may of course be observed. A slight correction would be theoretically required as an allowance for orbital motion of the earth; the effect of which would tend to increase the time, and which would be accordingly corrected by deduction. The quantity, however, would be extremely minute (about the 1400th of a degree of orbit,) and insufficient to be appreciable in the practical application of the method.

than either of the methods previously described or hitherto practised.

To illustrate this method: let the angle of the sun's semi-diameter viewed from the station be assumed as 16′, and viewed from the earth's centre as 15′ 59″·96, the difference of one-twenty-fifth of a second on the earth's radius, taking the metrical value of that radius, as before, at 4,000 miles, would give about 96 million miles as the sun's distance.* In the figure (fig. 8) the enormous exaggeration of the ratio of the earth's radius to the sun's distance (by representing the sun near to the earth) renders the basis of the method more distinctly apparent.

* The computation is . . $60 \times 25 \times 16 = 24{,}000$; which $\times\ 4{,}000 =$ 96 millions.

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